Robust Reference Tension Optimization in Winding Systems
Using Wound Internal Stress Calculation

M’hamed BOUTAOUS(1), Patrick BOURGIN(1), Dominique KNITTEL(2)

(1) Centre de Thermique de Lyon (CETHIL), INSA, Bâtiment Sadi Carnot,
20 Av. Albert Einstein, 69621 Villeurbanne Cedex, France

(2) University of Strasbourg, Institute of Physics and Engineering,
17, rue du Maréchal Lefèbvre, 67100 Strasbourg, France

mhamed.boutaous@insa-lyon.fr       patrick.bourgin@ec-lyon.fr   knittel@unistra.fr

ABSTRACT
It is well known that the tension reference value, which a priori guarantees a good quality roll, is based on the stress generated within the roll. The tension reference is optimized by considering both the tangential and the radial stress within the roll during winding. Moreover, a dynamic stress gauge is introduced, so that it can vary during the winding process. It generally represents the limits of elastic deformations of the web. A second approach consists to optimize the tension reference gauges, i.e. maximum and minimum reference values around the optimized nominal reference curve.

1 INTRODUCTION
From mathematical models of the stress within a roll, a method is proposed for computing an optimized winding reference tension, which is used to control the whole winding chain. The control of such systems is generally based on practical experiences and the tension reference curve does not change during winding: see for example Reid et al. [1], or Wolfermann [2] and [3].

The robust control approaches, synthesized with $H_\infty$ optimization (for example robust PI controllers), has been recently studied by Knittel et al. [4], [5], [6], [7], [18], [19]. All these control strategies can be considered as “offline tension control” based on classical winding models. As described in Boutaous et al. [10], the optimized winding tension is obtained by the simplex principle [11] which is a standard optimization algorithm, or by genetic algorithms. The tension reference is computed and corrected for each range of roll radius values, by using the predictive model for the stresses within the roll. The adjusted tension is re-actualized step by step, following the optimization principle and it will be considered as the new tension reference value for the coming layers.

2 WINDING TENSION OPTIMIZATION
Offline optimization of the tension reference is supposed to guarantee a priori the winding of a perfect roll. However, in reality, the control strategy never generates perfect follow-up of the reference. The applied tension thus does not lead any more to the optimal stress state in the roll. Therefore, it is consequently judicious to change the tension reference for the layers which still remain to be wound, throughout all the phase of winding, in order to always optimize the stresses in the final roll. The principle of this online approach is sketched in figure 1, where the roll is divided into a number $N$ of packs of layers.

To optimize the nominal winding tension reference, a mathematical model of stress computation is used to define a criterion $J$:

$$J = \min_{R_{\text{max}}^{T}, R_{\text{max}}^{R}} (J_T, J_R)$$

where:

$$J_T = \int_{R_{\text{rol}i}}^{R_{\text{rol}f}} (\sigma_T(T_w) - \sigma_T^{\text{mean}}(r))^2 g(\sigma_T, r) dr$$

$$J_R = \int_{R_{\text{rol}i}}^{R_{\text{rol}f}} (\sigma_R(T_w) - \sigma_R^{\text{mean}}(r))^2 g(\sigma_R, r) dr$$

(1)
and
\[
J_R = \int_{R_{inf}}^{R_{max}} \left( \sigma_R(T_w) - \sigma_{R\text{mean}}(r) \right)^2 g(\sigma_R, r) dr
\]
(3)

\(T_w\) represents the winding tension, \(\sigma_T\) is the tangential stress and \(\sigma_R\) radial stress. The stresses are calculated using the stress state mathematical model, Bourgin et al. [12], Connolly et al. [13], or Hakiel’s [14] one for instance.

\(\sigma_{\text{mean}}\) is some averaged tangential or radial stress value, in a given stress range (stress gauge) and \(g(\sigma, r)\) denotes some penalty function defined by:

\[
g(\sigma_T, r) = 1 \quad \text{if} \quad \sigma \quad \text{is in the gauge}
\]

\[
g(\sigma_T, r) \quad \gg \quad 1 \quad \text{else.}
\]

The reference tension which minimizes the cost function \(J\) is optimized using an algorithm based on the principle of the simplex presented by Nelder and Mead [11]. The criterion \(J\) is calculated for both the tangential and radial stresses, and the minimum one gives the optimal control tension.

The gauge limiting the range of acceptable stress fluctuations is fixed in the offline approach. The gauge can be variable during the optimization process, and generally it is chosen so that to respect the limits of the elastic deformation of the wound web.

![Diagram](image)

Figure 1: Online optimization of the winding tension reference

3 COMPARISON BETWEEN THE OFFLINE AND ONLINE CONTROL WITH A CONSTANT GAUGE

3.1 Perfect control:
The optimization of the winding tension for an ideal tension control, means that we have no disturbance, and the measured tension is identical to the reference (in our studied case, a decreasing reference line according the roll radius is used). For the comparison, we assume that the gauge is the same one in offline and in online optimization, and this gauge is fixed during all the process of winding.

In figure 2, one can see that the iterative optimization (online) gives the same control tension as that obtained offline. This result is expected: in our example it is assumed that measures and reference are the same. For the calculated stress state, using the online or the offline optimization, we have no differences too. Consequently, the online optimization does not make improvement in the case of a perfect system. Its contribution appears only in the case of a system subjected to disturbances.
3.2 Control with perturbations in tension measurement:
In the real case, we are always faced to disturbed tensions: the measured tension is different from the reference one. Due to the important role which the tension plays in the stress state within the roll during winding, see Pfeiffer [15], it is important to reject the deviations or noise by adjusting the reference tension during the winding process. In figures 3 (3a, 3b, 3c), we can observe how the random noise perturbations in the control tension affect both the radial and the tangential stress state.

![Figure 3a: Offline, real and optimized control tension with random noise](image)

![Figure 3b: Radial stresses](image)

![Figure 3c: Tangential stresses](image)

Figure 3: Offline and online optimized stresses, with static gauge.
From these figures we can give two important remarks:

1. The tangential stresses are more affected than the radial stresses by the tension disturbances.
2. The stress state calculated using the offline control is different from that one calculated by mean of the online control tension.

For this example we apply a constant stress gauge during all the phase of optimization. Figure 3a shows the comparison between the offline tension, and the tension calculated using our optimizing method, which smoothes the perturbation and suggests another law of the reference tension, corrected after each wound layer. This correction and prediction step by step of the evolution of the reference tension, can give us the adequate stress state, in sense of respecting the stress gauge values.

We can see that the online optimized reference curve is close to the “offline curve” at the beginning of winding, because the disturbances have affected only the already wound part (low number of layers), for the prediction (the remainder of the roll) we assume that there are no disturbances. As we continue the winding, the online calculated instruction deviates from that offline one. In figure 3c, various stress states are represented during the roll winding (real measurements for the already wound part, and prediction with ideal tension for the remainder to be wound). Curve 1 represents the tangential stresses of the total fictitious roll at the beginning of the process (the control tension is the offline one), and the curve 2 represents the tangential stresses at the end of winding. We observe that the perturbation affects only the wound part, but not the remainder part.

4 COMPARISON BETWEEN THE OFFLINE AND ONLINE CONTROL WITH A DYNAMIC GAUGE

A constant gauge is not always sufficient to have a good optimization. A dynamic variable stress gauge should be applied. For example, in the central zone of the roll, we can have a negative stresses, and the quality of the roll will be altered. One solution is to avoid this situation, by imposing a strong condition in the optimization process, by changing the value of the gauge in this zone. It is what we call, a dynamic gauge. In the figures 4 (a, b, c) we have illustrated these situations, by assuming periodic simulated perturbations. We have represented the evolution of the control tension and the stresses within the roll in case of imposing a constant stress gauge for the tangential stresses and a variable one for the radial stresses. In the central roll zone, the lower limit of the radial gauge is less important than in the remainder part of the roll. The result is that the tangential stresses values in this zone are upper than that obtained by means of a constant gauge and online optimization, or by means of an offline control.

![Graph showing control tension evolution](image)

Fig. 4 a: Offline and optimized control tension, with dynamic gauge
5 EFFECT OF THE REFERENCE TENSION VARIATIONS RATE DURING WINDING

Usually, the tension reference is deduced from industrial know-how: see for instance Reid et al. [1], Wolfermann [2] or Boutaous et al. [10]. The control of winding systems is generally based on practical experience and the tension reference does not change or decreases according to a more or less complicated function of the radius.

In this paper, for didactics reason, we assume that the winding tension decreases linearly versus the radius. It is clear that the approach described in this paper can be applied to more complex functions. In our example, we will show that the slope plays also an important role.

The optimum tension is that which guarantees that the stresses within the roll still confined in a desired stress gauge. But for industrials, the important and practical question is to know the limits of variations of the reference tension.

In other words, the idea is to find the maximum and minimum gauge for the reference tension, so that all curves ranging between these two limits (thresholds) generate radial and tangential stresses, included in a gauge fixed in advance. The problem is illustrated in the following example, see figure 5:

One winds a roll with a reference tension which decreases linearly versus the radius. This linear tension has to be optimized. It is represented by the line equation: \( T_{\text{opt}} = a.R + b \), where \( a \) and \( b \) have been optimized so that a criterion \( J \) is minimum, see [10].

As an example, let’s choose two tension gauges: \( T_{\text{max}} = T_{\text{opt}} + 1 \) and \( T_{\text{min}} = T_{\text{opt}} - 1 \), and also choose two other curves included inside the two gauges: therefore \( T_1 \) varies from 20 N to 4 N and \( T_2 \) varies from 18 N to 6 N. As these two curves (winding tension references) are located inside the two limits, one could expect that the corresponding calculated stresses curves are also confined between the stress calculated with \( T_{\text{max}} \) and
those calculated with $T_{\text{min}}$.
To compute the stresses state within the roll being wound, a modified non-linear model is developed in the spirit of Hakiels's [14]. Using the linear tension reference defined bellow, the tangential an radial stresses are calculated. To make the comprehension easy, only the tangential stresses are represented.
Surprisingly, the obtained results are different from that expected, as illustrated in figure 6. Indeed, the stresses $\sigma_T(T_1)$ calculated with $T_1$ as reference, is not confined between $\sigma_T(T_{\text{max}})$ and $\sigma_T(T_{\text{min}})$. The same observation has been made for tangential stresses $\sigma_T(T_2)$ computed with $T_2$ as reference tension. We can conclude that the slope of the reference tension influences the stresses as well as the tension itself. The magnitude of the tension is a key parameter, but not sufficient. The rate of the tension decrease plays an important role too. In fact, in figure 6, the stress state calculated with the reference tension ($T_{\text{max}}$) greater than $T_1$ or $T_2$, leads to stresses lower than that calculated with $T_1$ and $T_2$ in a certain range of radius, and inversely in other radius range.

![Figure 6](image)

> Figure 6 - Tangential stresses for several reference tensions.

**OPTIMISATION OF THE ADMISSIBLE TENSION DISPERSION**

We assume that the reference tension remains at first constant and after a certain value of the roll radius it decreases linearly: $T_{sl} = a.r + b +/- c$, where $c$ is a parameter introducing a dispersion about the nominal value of $T_{sl}$. The parameter $c$ is weighted by a function $F(r)$ which depends on the radius and beforehand selected, figure 7.

The same algorithm based on the theory of Nelder and Mead [11], is used to optimize the parameter $c$ of the tension dispersion, in order to minimize the difference between the stress gauge and the lowest value calculated for the tangential stresses.
In the real case, the dispersion has never a constant value, but changes during winding. To simulate this effect, one take a reference tension shown previously in figure 7a, but this time we assume that the parameter $c$ increases when the roll radius increases as in figure 8a. The results are shown in figure 8b. The results show that the tension rate (slope of the curve) is an important parameter, but the type of the fluctuations (stable or varying) plays an important role too.
This case is particularly useful, when one does not know the slope to be chosen for a decreasing reference tension.

6 CONCLUSION

Because the web tension acts directly on the stress state within a roll, it is very important to master and to optimize it during the winding process. To do, a mathematical model calculating the stress state within a roll is used to optimize the tension reference during the roll winding. An optimization strategy called “online optimization” is used and compared to the classical “offline” one. The comparison shows interesting improvements, in terms of insuring an internal stress state compatible with elastic deformations of the web within the roll.

A second approach consists to optimize the tension reference gauge in order to make the choice of the winding reference confined in a constraining gauge, with respect to the stress state within the roll.
7 REFERENCES


