Elastic web processing lines : optimal tension and velocity closed loop bandwidths

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This paper concerns the tension control of roll-to-roll systems. In such systems, each driven roller speed is controlled separatly. The impact of the master roller position on tension control performances is studied using optimized web tension control synthesis. The tension disturbance rejection property and the system robustness regarding web elasticity variations are analysed for each configuration. Then the influence of the tension and velocity bandwidths is studied.

1 Introduction

Systems handling web materials such as textile, paper, polymer or metal are very common in industry, because they represent a convenient way of transporting and processing a product from one roll to another. The variables that need to be monitored and controlled in order to achieve the expected product quality are web tension and speed. In order to control the web tension, a master driven roller has to be chosen. The choice of the master roller has an influence on the tension control structure.

To study the impact of the master driven roller position, a roll-to-roll system composed of 7 driven rollers and 6 idle rollers is studied. A linear model of this plant is developed for the automatic controller synthesis in the Matlab/Simulink software environment [1]. Each driven roller speed is controlled by an IP controller, whereas the web tension is controlled using PI controllers.

Figure 1: Roll-to-Roll plant at University of Strasbourg

2 Modelling

The model of a web transport system is built from the equations describing the velocity of each roller and the web tension behaviour between two consecutive rollers [1], [2], [12].

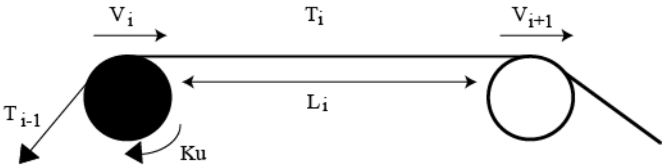

Figure 2: web span

2.1 Web speed determination

The speed of a roller V_i, which depends on the upstream web tension T_{i-1} and the downstream web tension T_i, is given by :

$$J_i \frac{dV_i}{dt} = (T_i - T_{i-1})R^2 + KRu_i - f_d V_i \tag{1}$$

where J_i is the roller inertia, T_i is the downstream web tension, T_{i-1} is the upstream web tension, K is the torque constant of the motor, R is the roller radius, u_i is the driven roller control signal and f_d is the dynamic friction coefficient. This equation assumes that no slippage occurs : the web speed is equal to the linear roller speed. Moreover the static frictions are neglected.

2.2 Web Tension determination

The strain ε of a web span, which depend on the upstream web strain and the downstream roller speed, is given by [1] :

$$\frac{d}{dt}\left(\frac{L_i}{1+\varepsilon_i}\right) = -\frac{V_{i+1}}{1+\varepsilon_i} + \frac{V_i}{1+\varepsilon_{i-1}} \tag{2}$$

where L_i is the web span length, V_{i+1} is the downstream roller speed, V_i is the upstream roller speed and ε_{i-1} is the upstream web strain.

The web tension T is obtained using Hooke's law :

$$T = ES\varepsilon \tag{3}$$

where E is the web Young modulus and S is the web cross-section.

The web tension is determined using the non-linear equation (2). This equation can be linearised around a working point T_0, V_0. Considering $T_i = T_0 + t_i$, $V_i = V_0 + v_i$, $T_{i-1} = T_0 + t_{i-1}$ and $V_{i+1} = V_0 + v_{i+1}$ the linear equation becomes:

$$L_i \frac{dt_i}{dt} = V_0(t_{i-1} - t_i) + (v_{i+1} - v_i)(ES + T_0) \tag{4}$$

2.3 Linear Model

The relations depicted in (1) and (4) permit to construct the state-space representation of the studied system:

$$\begin{cases} \dot{x}(t) & = & \mathsf{A}x(t) + \mathsf{B}u(t) \\ y(t) & = & \mathsf{C}x(t) + \mathsf{D}u(t) \end{cases} \tag{5}$$

where x is the state vector, u the control vector, y the output vector. A is the state matrix, B the input matrix, C the output matrix and D the feedforward matrix.

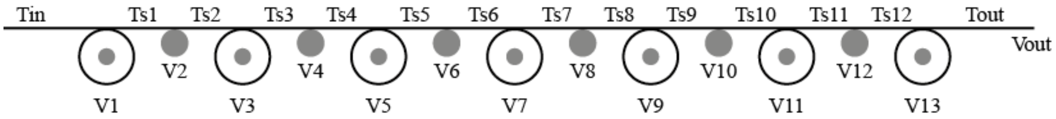

Figure 3: System under study

The system scheme is shown in Fig. 3. To model such a system, the speed of each roller and the tension of each web span have to be calculated. The state vector is composed of the velocity of each driven roller, the velocity of each idle roller (equipped with a load cell) and the web tension in each web span.

$$x = [V_1 \ T_{s1} \ V_2 \ T_{s2} \ V_2 \ T_{s3} \ \cdots \ T_{s12} \ V_{13} \ T_{out} \ V_{out}]^T \tag{6}$$

The system has 9 entries : the system input web tension, the system output web speed and the 7 motor control signals.

$$u = [T_{in} \ V_{out} \ u_1 \ u_2 \ u_3 \ u_4 \ u_5 \ u_6 \ u_7]^T \tag{7}$$

The system has 13 outputs : the system driven roller rotational speeds and the web tension located at each idle roller.

$$y = [\omega_1 \ \omega_2 \ \omega_3 \ \omega_4 \ \omega_5 \ \omega_6 \ \omega_7 \ T_1 \ T_2 \ T_3 \ T_4 \ T_5 \ T_6]^T \tag{8}$$

where :

$$\begin{aligned} T_1 &= \frac{Ts1 + Ts2}{2} \quad T_2 = \frac{Ts3 + Ts4}{2} \quad T_3 = \frac{Ts5 + Ts6}{2} \\ T_4 &= \frac{Ts7 + Ts8}{2} \quad T_5 = \frac{Ts9 + Ts9}{2} \quad T_6 = \frac{Ts11 + Ts12}{2} \end{aligned} \tag{9}$$

3 Control strategy

In Roll-to-Roll systems, two variables have to be mastered: the web speed and the web tension in each span. To control these variables, cascading control structure can be used as shown in Figure 4. First C_C is used to control the motor torque. This control loop is very fast. Then C_V is used to control the roller speed. Finally, C_T is used to control the web tension. The roller speed and web tension control loop are described more precisely in the following sections.

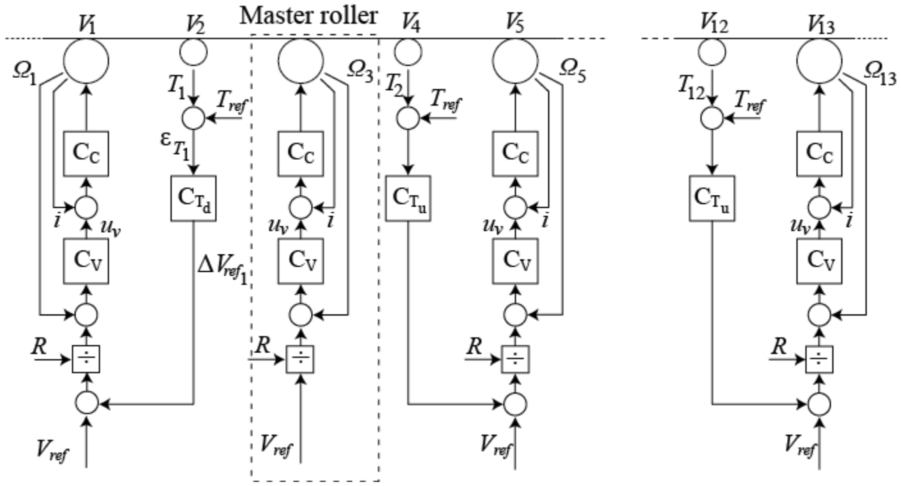

Figure 4: Example of a decentralized control strategy with the master driven roller in second position

3.1 Speed Control

The speed control is ensured using IP controller. The main asset of the IP controller is that it does not introduce a zero in the closed-loop transfer function.

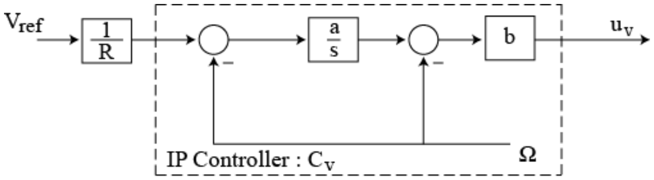

Figure 5: IP speed controller

The speed controller scheme is given in Fig. 5. The linear speed reference is divided by the roller radius in order to have a rotational speed reference. The IP controller parameters a and b are calculated as follows:

$$a = \frac{\omega_n}{2\zeta} \qquad b = \frac{\omega_n^2}{a.K} \tag{10}$$

where ω_n is the desired closed loop cross-over frequency, ζ is the desired damping factor and K is the motor torque constant. In fact, when the friction is neglected, the motor transfer function is equal to :

$$\frac{\Omega}{u} = \frac{K}{Js} \tag{11}$$

where Ω is the rotational roller speed, u is the motor control signal, J is the roller inertia. The torque closed loop is considered as perfect and approximated by a gain K .

3.2 Tension Control

For the tension PI controller synthesis, a H_∞ approach is used [5], [6], [11]. The H_∞ problem formulation is to find a stabilizing controller that minimise the H_∞ norm of the transfer function between a set of exogenous inputs w and a set of performance outputs z .

$$\left\| T_{w \to z} \right\|_\infty < \gamma \tag{12}$$

4 Influence of master driven roller position

To control the web tension, the position choice of the master driven roller has an important impact on the system performances. To describe the impact of this choice, three configurations are compared. The first configuration uses the second driven roller as master roller. The second configuration uses the central driven roller as the master roller. Finally, in the last configuration the sixth driven roller is the master roller.

These three configurations are compared together using a time based and a frequency based analysis. In Roll-to-Roll systems, the most important characteristics to ensure are the web tension reference tracking, the web tension perturbation rejection and the robustness regarding web elasticity variations.

4.1 Reference Tracking Performance

Web tension reference tracking is a key point in Roll-to-Roll systems. The resulting tension in each web span is then compared for the three position configurations of the master driven roller (a step is applied on the web tension reference).

The web tension simulations are presented in Fig. 6. One can see that the tensions close to the master driven roller have the best performances. The web tension reference tracking performance decreases when it keeps away from the master roller. The best master roller position depends on the requirements on the system. The master roller has to be located close to the area in which the web tension has to be well mastered.

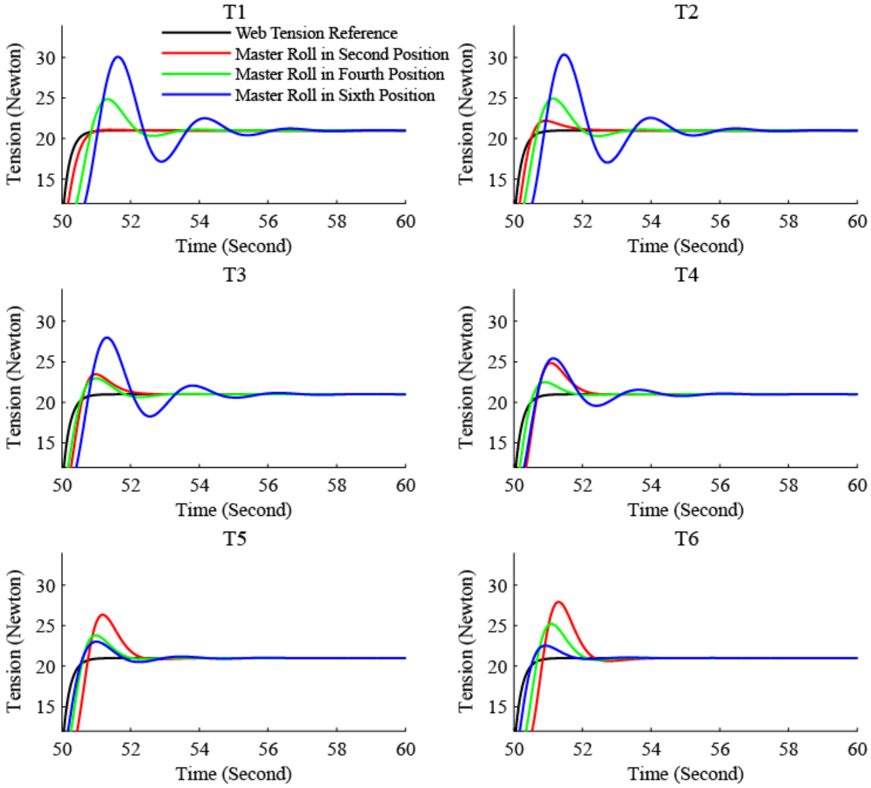

Figure 6: reference tracking : comparison of the three configurations

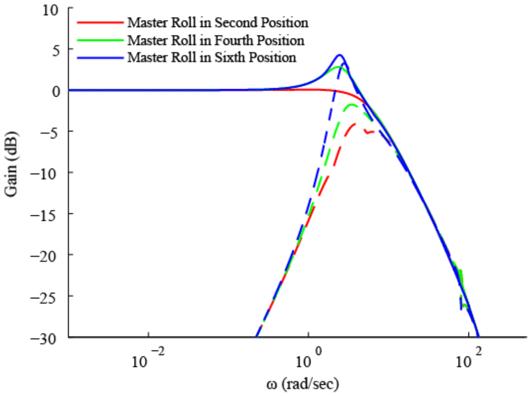

Figure 7: Maximum of the diagonal transfer function (solid) and non-diagonal transfer function (dashed) : comparison for the three configurations

The Bode diagram in Fig. 7 shows the maximum diagonal and non-diagonal transfer functions of the closed-loop system (a diagonal transfer function is defined as a transfer function between a tension reference $Tref_i$ and the corresponding measured web tension T_i. A non-diagonal transfer function is defined as a transfer function between a web tension reference $Tref_i$ and another measured web tension T_j). In fact, the maximum non-diagonal transfer function shows the maximum coupling between tensions. The frequency study shows that the best system is the system with the second driven roller as master roller. In fact, this configuration have the minimum coupling transfer function and the lowest H_∞ norm of the diagonal transfer function. The configuration with the sixth driven roller as master roller seems to be the worst case, it has the maximum coupling between the tensions and the worst H_∞ norm. These results are validated by the simulations in Fig. 6.

4.2 Disturbance Rejection

In Roll-to-Roll systems, the web tensions are disturbed by the roll eccentricity and non-circularity. The eccentricity involves a sinusoidal web tension disturbance with a frequency equal to the roll rotational frequency. The non-circularity involves a periodic signal composed of a fundamental sinus and different harmonics with different amplitudes.

In this study, the disturbance signal is composed of the sum of 4 sinus with the same amplitude, in order to compare the attenuation in each frequency range.

To compare the master roller position in terms of disturbance rejection, the disturbance signal is applied on different web tensions along the processing line. First the disturbance signal is added on T_{in}.

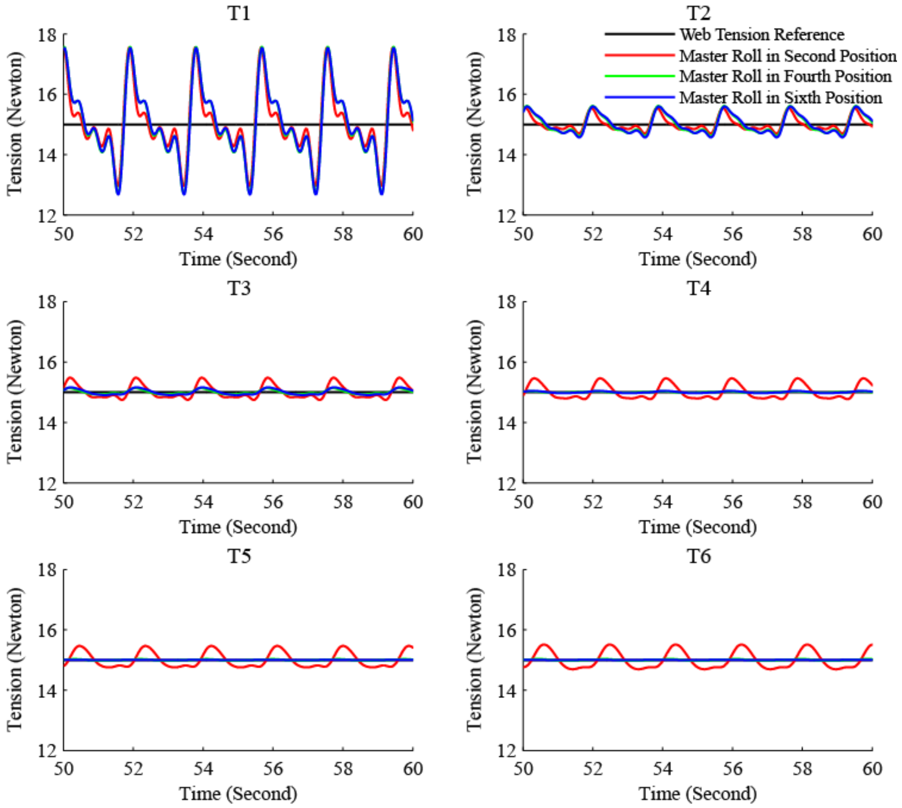

Figure 8: Simulation with a disturbance on Tin

The simulation results are shown in Fig. 8. The configuration with the master driven roller in second position has the worst performances. The two other configurations have very similar results.

Then the disturbance is applied on the intermediate tension. For instance, in Fig. 9 the disturbance is applied on Ts_5.

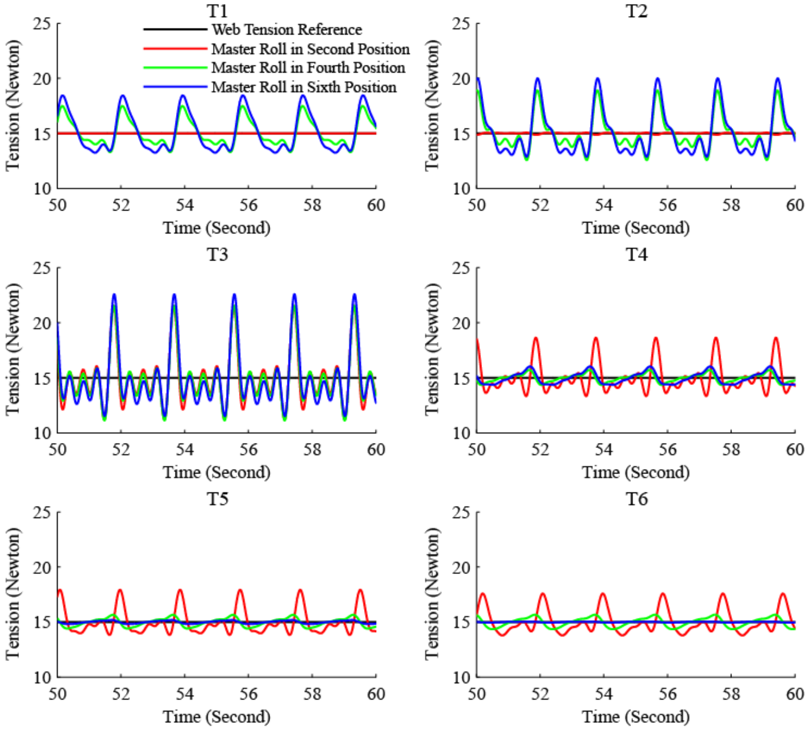

Figure 9: Simulation with a disturbance on Ts_5

4.3 Robustness regarding web elasticity variation

Web elasticity usually varies with air temperature and moisture. This variation can decrease the system closed loop performances. Moreover some process lines include systems which can make the web elasticity varying.

In order to evaluate the robustness of the master roller configurations, the web elasticity is divided by two and the three system configurations are compared.

The simulation results are depicted in Fig. 10. The configuration with the master driven roller in sixth position is the most sensitive to the web elasticity variation. On the configuration with the master roll in fourth position the performance does not decrease significantly. Finally, the configuration with the master roller in the second position is the most robust because its performance is not decreasing when the web elasticity is varying.

Master roller in	Mean of reference tracking error	Disturbance rejection	Robustness regarding E Variation
Second Position	+++	+	+++
Fourth Position	++	+++	++
Sixth Position	+	+++	+

Table 1 : comparison of the three configurations

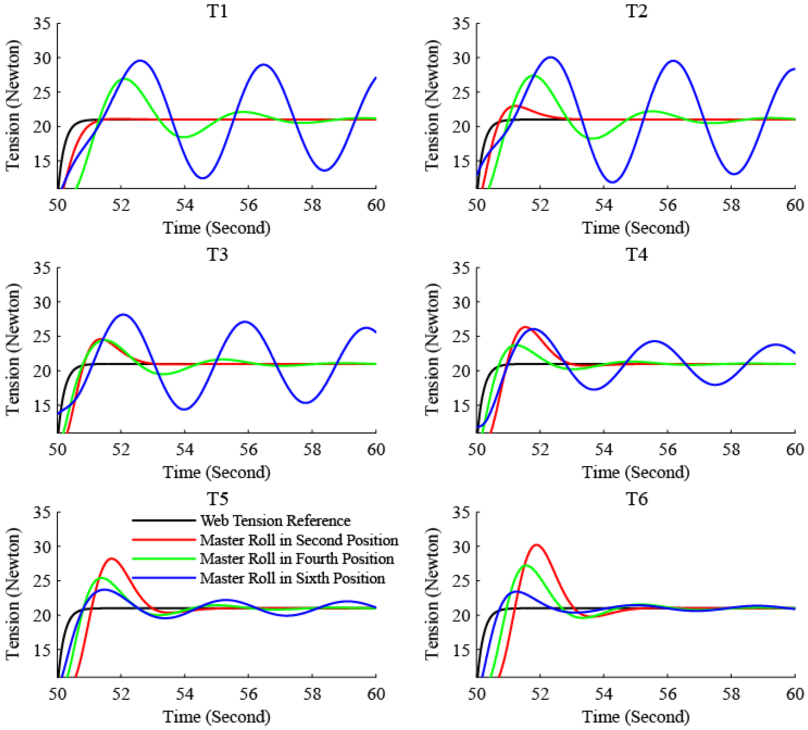

Figure 10: Simulation with a reduced web elasticity

4.4 Influence of the tension and velocity closed loop bandwidths

In cascading control, the internal loop behaviour has an important effect on external loop performances. In this application, the speed loop can be considered as a second order system, its crossover frequency acts on the web tension loop performances as shown in Fig. 11, 12, 13, 14.

Fig 12 and 14 show the robustness analysis to reduced web elasticity (E_{nom}/100).

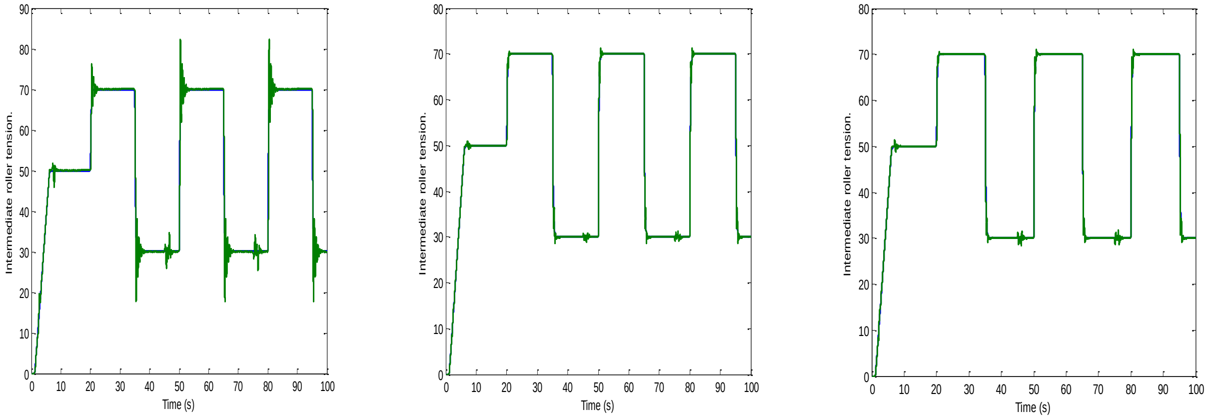

Figure 11: Web tension for a tension bandwidth 5rad/s and different velocity bandwidths (15rad/s, 30rad/s, 50rad/s)

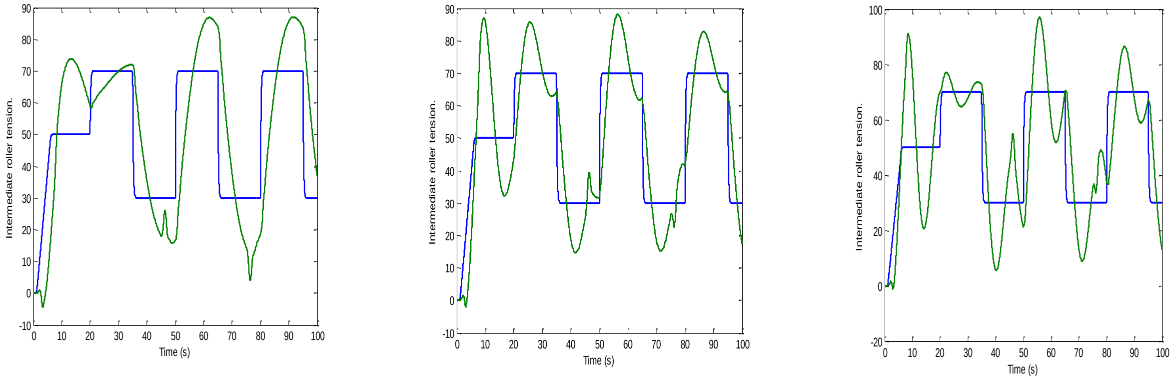

Figure 12: Simulations with reduced web elasticity (E_{nom}/100) : Web tension for a tension bandwidth 5rad/s and different velocity bandwidths (15rad/s, 30rad/s, 50rad/s)

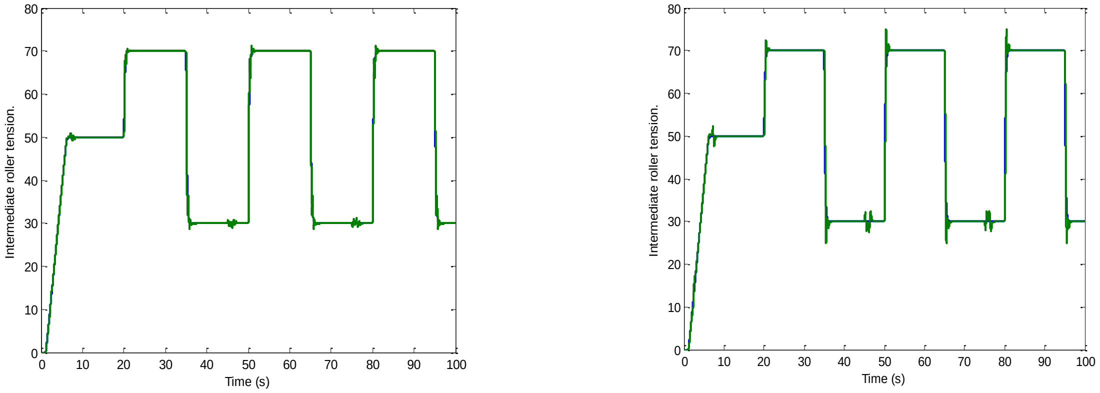

Figure 13: Web tension for a velocity bandwidth 30rad/s and two tension bandwidths (5rad/s, 15rad/s)

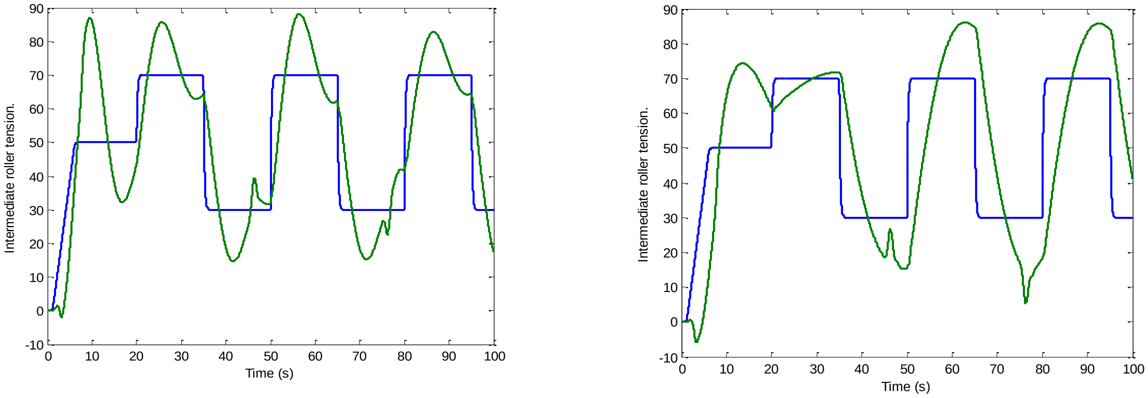

Figure 14: Simulations with reduced web elasticity (E_{nom}/100) : Web tension for a velocity bandwidth 30rad/s and two tension bandwidths (5rad/s, 15rad/s)

A lower velocity crossover frequency implies better web tension reference tracking performance and better robustness to web elasticity variations. In fact, the speed loop acts as a second order filter on the web tension loop : if the filter crossover frequency is low (or not too high) the tension response is highly filtered. Simulations have shown that the tension closed loop bandwidth should be smaller than the velocityclosed loop bandwidth.

5 Conclusion

This paper shows the importance of the master driven roller position in Roll-to-Roll systems. It is shown that the choice of the master driven roller highly depends on the use of the system. In fact, the master driven roller has to be placed close to the web span where highest performances are needed.

However when a robustness to web elasticity variations is needed, the master roller should be placed at the beginning of the line. The effects of the tension and velocity closed loop bandwidths are also highlighted.

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