The Simple Math of Web Handling

Web Handling
And
The Power of Modeling

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The Simple Math of Web Handling

- What is web handling modeling?
  - At its simplest, it is using basic formulas to do calculations and make decisions.

- The basic formulas are the foundation for first design, changes, and troubleshooting.

- Everyone involved in web handling should be familiar with and using the basic math.
Basic Web Handling Formulas

- Strain Transport Formula
- Modulus Formula
- Idler Drag Formula
- Capstan Formula
- Neckdown Formula
Basic Web Handling Formulas

- **Strain Transport Formula**

\[
\frac{V_n}{1 + \varepsilon_n} = \frac{V_{\text{ref}}}{1 + \varepsilon_{\text{ref}}}
\]

Load Cell or Dancer
Basic Web Handling Formulas

- Modulus Formula

\[ T_n = \varepsilon_n E_w \]

Tension and modulus are in force units. This is ideal for transport calculations.

\[ E_w = E A \]

if \( E = 32 \text{MPa}, \ h = 0.00025 \text{ m}, \ W_0 = 2.5 \text{ m} \)

then

\[ E_w = E A = 32,000,000 \times 0.00025 \times 2.5 = 20,000 \text{ N} \]
Combining The Strain Transport and Modulus Formulas

\[ T_n = E_w \left( \frac{V_n}{V_{ref}} \left( 1 + \frac{T_{ref}}{E_w} \right) - 1 \right) \]

\[ T_8 \text{ depends only on } T_{ref}, V_{ref}, E_w \text{ and } V_8 \]

\[ \varepsilon_n = \left( \frac{V_n}{V_{ref}} \left( 1 + \frac{T_{ref}}{E_w} \right) - 1 \right) \]
Basic Web Handling Formulas

- Idler Drag Formula
  (If in traction)

\[ D_n = T_n - T_{n-1} \]
\[ T_{n+1} = T_n + D_n \]

Idler drag is the force applied at the idler surface necessary to keep the idler spinning at nominal velocity. Idler drag is in force units.
Basic Web Handling Formulas

- Capstan Formula

To maintain traction with rollers

\[
\frac{T_h}{T_l} < e^{\mu \theta}
\]

- $\theta$ is the wrap angle in radians
- $\mu$ is the coefficient of friction
- $T_l$ is low side tension
- $T_h$ is high side tension
Basic Web Handling Formulas

- Capstan Formula

To maintain traction with rollers

\[ \theta = \pi \text{ radians} \]
\[ \mu = 0.3 \]
\[ T_l = 15 \text{ N} \]
\[ T_h = 50 \text{ N} \]

\[ \frac{T_h}{T_l} < e^{\mu \theta} \]
\[ \frac{50}{15} < e^{0.3 \times \pi} \]

3.33 \( \neq \) 2.56

Roller Will Slip!
Basic Web Handling Formulas

- **Capstan Formula**

\[
\frac{T_h}{T_l} = e^\mu \theta
\]

For sliding contact

- \( \theta \) is the wrap angle in radians
- \( \mu \) is the coefficient of friction
- \( T_l \) is upstream tension
- \( T_h \) is downstream tension
Basic Web Handling Formulas

**Neckdown Formula**

\[ W_n = W_0 - \frac{T_n}{N_m} \]

\[ N_m = \frac{\Delta T}{\Delta W} \]

A 0.2 m wide web has 4 mm of neckdown at 20 N tension.

\[ N_m = \frac{\Delta T}{\Delta W} = \frac{(20 - 0)}{0.200 - 0.196} = 5000 \text{ N/m} \]

The same material with \( W_0 = 2.5 \text{ m} \) and \( T_n = 50 \text{ N} \), will have 10 mm of neckdown.

\[ W_n = W_0 - T_n \left( \frac{T_n}{N_m} \right) = 2.5 - \frac{50}{5000} = 2.49 \text{ m} \]
Lamination Path Example

What do we need to run a new cost savings material on one ply?

Current material runs well. No curl.

\[
\begin{align*}
E &= 32 \text{MPa Current material} \\
W_0 &= 2.5 \text{ m} \quad h = 0.00025 \text{ m} \\
E_w &= 32 \text{MPa} \times 2.5 \times 0.00025 \\
E_w &= 20,000 \text{ N Current material}
\end{align*}
\]

\[
\begin{align*}
D_n, \Delta T \text{ across idlers} &= 5.0 \text{ N} \\
T_1 &= 20 \text{ N for successful splice} \\
\varepsilon_1 &= 0.001 \\
T_3 &= 30 \text{ N}
\end{align*}
\]

Load cell adjusted so \( T_3 = 30 \text{ N} \)

New material: \( E_w = 10,000 \text{ N} \)
Lamination Path Example

Alternate #1: Hang and Run

Results: The loadcell will still maintain $T_3$ at 30 N. Adding 5 N for the drag of each idler downstream we find that $T_{13}$ is still 80 N.

Now however, with the lower modulus material, $\varepsilon_{13} = 0.008$.

Conclusion: There will probably be MD and CD curl due to the strain mismatch. This is not a good alternative.
Lamination Path Example

**Alternate #2:**
Lower loadcell setpoint to achieve $\varepsilon_{13} = 0.004$ strain at lamination

**Results:** At $\varepsilon_{13} = 0.004$ strain at lamination, the new material generates $T_{13} = 40$ N of tension in span 13. Subtracting 5 N from $T_{13}$ tension for each idler shows tension will be zero in span 5.

**Conclusion:** This will not run due to no tension at the parent roll.

\[ T_1 = 0 \text{ N} \]
\[ \varepsilon_1 = 0.0 \]
\[ T_6 = 5 \text{ N} \]
\[ T_5 = 0 \text{ N} \]
\[ V_{13} \]
\[ T_{13} = 40 \text{ N} \]
\[ \varepsilon_{13} = 0.004 \]
Lamination Path Example

Alternate #3: Add a mid-point driven roller to help pull web

Keep the unwind tension at $T_1 = 20$ N, then $T_7$ needs to be 50 N.
Keep the combining strain at $\varepsilon_{13} = 0.004$, then $T_{13} = 40$ N and $T_8 = 15$ N.
Lamination Path Example

Alternate #3: Add a mid-point driven roller to help pull web

The velocity of $V_7$, can be calculated using the strain transport formula, with $\varepsilon_7$, $\varepsilon_{13}$ and $V_{13}$ as inputs.

\[
\frac{V_7}{1 + \varepsilon_7} = \frac{V_{13}}{1 + \varepsilon_{13}}
\]

\[
V_7 = V_n \frac{1 + \varepsilon_7}{1 + \varepsilon_{13}}
\]

\[
V_7 = V_{13} \frac{1.005}{1.004}
\]

\[
V_7 = 1.001V_{13}
\]

- $T_1 = 20 \text{ N}$
- $\varepsilon_1 = 0.002$
- $T_2 = 25 \text{ N}$
- $T_3 = 30 \text{ N}$
- $T_7 = 50 \text{ N}$
- $\varepsilon_7 = 0.005$
- $T_8 = 15 \text{ N}$
- $T_{13} = 40 \text{ N}$
- $\varepsilon_{13} = 0.004$

Load Cell or Dancer
Lamination Path Example

Alternate #3: Add a mid-point driven roller to help pull web

Result: We get the correct strains and tensions, but the tension ratio, \( T_h / T_l = 50 / 15 = 3.33 \). This exceeds the 2.56 maximum and will slip. The large tension drop may also cause wrinkle problems.

Conclusion: A higher COF coating on the roller might solve the slip problem, but not the wrinkles problem. This web path is not a good alternative.
Lamination Path Example

Alternate #4: Add Two Driven Rollers

Keep $T_1 = 20$ N, then $T_5 = 40$ N. If $T_{13}$ must remain 40 N, then $T_{10} = 25$ N. The velocity of the second driven roller can be set to give any tension for $T_9$.

The positions and velocities of the two new driven rollers are used to balance the peak tensions and tension drops to minimize the chance of slip or wrinkles.
Lamination Path Example

Alternate #4: Add Two Driven Rollers

One of the realities shown by the strain transport formula is that if two rollers have the same velocity, they will have the same strain in the approaching span.

Since I want rollers 5 and 9 to have the same approaching strain as roller 13, they should have the same velocity as roller 13.

\[ T_1 = 20 \text{ N}, \quad \varepsilon_1 = 0.002 \]
\[ T_2 = 25 \text{ N} \]
\[ \text{Load Cell or Dancer} \]
\[ \text{Setpoint} = 27.5 \text{ N} \]
\[ T_3 = 30 \text{ N} \]
\[ T_5 = 40 \text{ N}, \quad \varepsilon_5 = 0.004 \]
\[ T_9 = 40 \text{ N}, \quad \varepsilon_9 = 0.004 \]
\[ T_{13} = 40 \text{ N}, \quad \varepsilon_{13} = 0.004 \]
Lamination Path Example

Alternate #4: Add Two Driven Rollers

\[ T_1 = 20 \text{ N} \]
\[ \varepsilon_1 = 0.002 \]
\[ T_2 = 25 \text{ N} \]
\[ T_3 = 30 \text{ N} \]
\[ T_5 = 40 \text{ N} \]
\[ \varepsilon_5 = 0.004 \]
\[ T_9 = 40 \text{ N} \]
\[ \varepsilon_9 = 0.004 \]
\[ T_{13} = 40 \text{ N} \]
\[ \varepsilon_{13} = 0.004 \]

**Result:** \( T_h / T_l = 40 / 25 = 1.6 \) which is less than the 2.56 maximum. This will give us a healthy safety factor to avoid slip during upset conditions. All rollers run at the same velocity as the lamination roller. My peak strains are the same as the lamination strain, so I am not causing any additional strain induced problems in the web from a high peak strain.

**Conclusion:** This path should run well. I need to test to determine if a 1.6 tension ratio will cause wrinkles.
Lamination Path Example

Additional Topics

- The loadcell measures the average tension of the two adjacent spans.
- The loadcell idler and the two idlers between it and the first driven roller are the three most important idlers in the web path.
- Relocate the loadcell to the last idler position prior to the first driven roller and changing the setpoint to 37.5 N.
- Drive a bowed axis roller the same velocity as the lamination roller. If it is undriven, they are poor idlers with high and unknown drag. I would drive the bowed roller if I could.
- The mathematics of web handling helps to identify the lowest cost alternative that is likely to work. It doesn't guarantee success, but it improves your odds.
- The cost savings must justify the cost of the changes.
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Thank You

Questions?

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